ПAmIBIA UחIVERSITY OF SCIEПCE AПD TECHПOLOGY

## FACULTY OF HEALTH, NATURAL RESOURCES AND APPLIED SCIENCES SCHOOL OF NATURAL AND APPLIED SCIENCES <br> DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

| QUALIFICATION: Bachelor of science ; Bachelor of science in Applied Mathematics and Statistics |  |
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| QUALIFICATION CODE: 07BSOC; 07BSAM | LEVEL: 5 |
| COURSE CODE: CLS502S | COURSE NAME: CALCULUS 1 |
| SESSION: JULY 2023 | PAPER: THEORY |
| DURATION: 3 HOURS | MARKS: 100 |


| SUPPLEMENTARY/SECOND OPPORTUNITY EXAMINATION QUESTION PAPER |  |
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| EXAMINER | Mrs. H.Y NKALLE |
| MODERATOR: | Dr. N. CHERE |

## INSTRUCTIONS

1. Answer ALL the questions in the booklet provided.
2. Show clearly all the steps used in the calculations.
3. All written work must be done in blue or black ink and sketches must be done in pencil.

## PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)

## Question 1 [2 Marks]

Consider the relation $\mathrm{P}=\{(1,7),(-1,7),(3,9),(1,3)\}$. Is P a function? Justify your answer.

## Question 2 [6 Marks]

Show that $\lim _{x \rightarrow 0} \frac{1-\cos x}{x}=0$.

## Question 3 [8 Marks]

Find $\lim _{x \rightarrow a} \frac{\sqrt{a+2 x}-\sqrt{3 x}}{\sqrt{3 a+x}-2 \sqrt{x}}$.

## Question 4 [5 Marks]

Find the average rate of change of the function $f(x)=x^{2}+4 x$ over the interval $[-6,4]$.

## Question 5 [5 Marks]

Find the instantaneous rate of change at $x=b$ for the function $f(x)=x^{2}+2 x$.

## Question 6 [8 Marks]

Consider the function $f(x)=e^{r x}$. Determine the values of $r$ so that $f$ satisfies the equation $f^{\prime \prime}(x)+f^{\prime}(x)-6 f(x)=0$.

## Question 7 [5 Marks]

Find the equation of the tangent line to the graph of the function $\mathrm{f}(\mathrm{x})=\sqrt{\mathrm{x}}$ at the point $(1,1)$.

## Question 8 [7 Marks]

Differentiate $f(x)=\frac{24 x}{6 x+5}$ from first principle.

## Question 9 [3 Marks]

Let $f(x)=c$. Find $f^{\prime}(x)$ using limit definition of derivative.

## Question 10 [5 Marks]

Find the range of $\mathrm{f}(\mathrm{x})=\sqrt{1-\mathrm{x}^{2}}$.
Question 11 [4 Marks]
Let $\mathrm{f}(\mathrm{z})=\ln \mathrm{z}$. Find $\mathrm{f}^{\prime \prime \prime}(\mathrm{z})$ at $\mathrm{z}=2$.
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## Question 12 [4 Marks]

Find $\frac{d y}{d x}$ by using implicit differentiation of $x y=1$.

## Question 13 [5 Marks]

Use logarithmic differentiation to find $\frac{\mathrm{d}}{\mathrm{dx}}\left(\mathrm{x}^{\sqrt{3}}\right)$.

## Question 14 [2, 2 Marks]

Investigate whether the following functions are odd or even.
(a) $f(x)=x^{3}$.
(b) $f(x)=\cos x$.

## Question 15 [9, 6 Marks]

Let $f(x)=\frac{1}{3} x^{3}+x^{2}-15 x-9$. Use detailed sign tables in answering the following questions.
(a) Find the intervals in which $f$ is increasing or decreasing.
(b) Find the intervals in which the graph of $y=f(x)$ is concave upward or downward.

## Question 16 [8 Marks]

Air is escaping from a spherical balloon at the rate of $2 \mathrm{~cm}^{3}$ per minute. How fast is the radius shrinking when the volume is $36 \pi \mathrm{~cm}^{3}$ ?

## Question 17 [6 Marks]

Find the rate of change of the area $A$, of the circle with respect to its circumference C, i. e $\frac{d A}{d C}$.

